Design of Frequency Architectures to Eliminate Mixer Spurious Responses

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Several different methods have been described in the literature to assess mixer performance for a given frequency architecture[1,2,3,4]. The most commonly used relationship shown in [1] is solved for those cases of M and N which give spurious outputs at the desired IF or within a range of the IF. This same relationship can be worked in reverse to identify different ranges within the IF which, when mapped through the mixer in reverse, identify input frequencies suffering from spurious mechanisms.

$$f_{out} = M\ f_{LO} + N\ f_{RF} \qquad [\ 1\ ]$$

Bain Technique for Mixer Characterization

The next several pages are intended to describe a method which is a derivative of [1] above, and was first published in rf Design, May, 1989[4]. I have filled in the missing details to help *document for myself* the technique and application of the algorithm. Two slightly different expressions related to [1] above are used to describe the desired and undesired signals appearing at the IF output.

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_x + M\ F_{LO} = F_{out} \qquad [\ 2, 3\ ]$$

The first expression, [2], describes the desired signal with N_1 and M_1 taking on values of ±1 to give an output at the IF. The second expression, [3], determines some frequency, F_x which, when combined with the spurious mechanisms of the mixer, also gives an output at the IF. F_x is defined in expression [4]. F_x is defined to be some signal slightly removed from F_{in} which, due to the spurious mechanism, produces undesired output at the IF. Δ_{Fx} describes this offset in frequency.

$$\Delta_{Fx} = F_x - F_{in} \qquad [\ 4\ ]$$

There are four different scenarios of interest with regard to frequency conversion schemes finding most use. The relationships we derive below eliminate an unknown swept variable, such as the local oscillator frequency in a fixed IF case, to culminate in the desired mathematical expression. This elimination technique is used throughout to synthesize relationships which show the position of the interfering signal within the IF passband.

Type 1 - Receiver Case	$N\ F_x + M\ F_{LO} = F_{out}$	F_x is Signal Offset From Desired Input Signal Giving Output for **Fixed IF**	$\Delta_{Fx} = F_x - F_{in}$ at Input F_{rfi} = Mapping of IF Passband Back to Input
Type 2 - Receiver Case	$N\ F_x + M\ F_{LO} = F_{out}$	F_x is Signal Offset From Desired Input Signal Giving Output for **Fixed LO**	$\Delta_{Fx} = F_x - F_{in}$ at Input F_{rfi} = Mapping of IF Passband Back to Input
Type 3 - Transmitter Case	$N\ F_{in} + M\ F_{LO} = F_x$	F_x is Signal Offset From Desired *Output* Signal Giving Output for **Fixed LO**	$\Delta_{Fx} = F_x - F_{out}$ at Output F_{rfi} = Mapping of Ouput Passband Back to Input
Type 4 - Transmitter Case	$N\ F_{in} + M\ F_{LO} = F_x$	F_x is Signal Offset From Desired *Output* Signal Giving Output for **Fixed RF** Input	$\Delta_{Fx} = F_x - F_{out}$ at Output F_{rfi} = Mapping of Ouput Passband Back to Input

1 "Normalized Lowest Intermod Mixer Bandwidth Design Curves," Neuf, D., Piro, P., Microwave Journal, February, 1985.
2 "A Computer Algorithm for Mixer Spurious Calculations," Victor, A., rf Design, July 1985.
3 "Graphing Spurious Responses in Microwave Receivers," Karpen, E.W., Mohr, R.J., Microwaves, Nov., 1966.
4 "A Mixer Spurious Plotting Program," Bain, R., rf Design, May, 1989.

Case 1: *Fixed Output, Swept Local Oscillator*

We begin with the following pair of equations:

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_x + M\ F_{LO} = F_{out} \qquad [5, 6]$$

Multiplying each equation by M or M_1 and subtracting to cancel F_{LO} gives:

$$\begin{array}{l} M\ N_1\ F_{in} + M\ M_1\ F_{LO} = M\ F_{out} \\ \underline{-\left(M_1\ N\ F_x + M\ M_1\ F_{LO} \right) = -\ M_1\ F_{out}} \\ N_1\ M\ F_{in} - N\ M_1\ F_x = F_{out}\left(\ M - M_1\ \right) \end{array} \qquad [\ 7\ ]$$

Making the substitution for F_x from [4] above gives:

$$N_1\ M\ F_{in} - N\ M_1\left(\ \Delta_{Fx} + F_{in}\ \right) = F_{out}\left(\ M - M_1\ \right) \qquad [8]$$

The input frequencies for F_{in} range from F_{min} to F_{max} which give two different answers for Δ_{Fx}.

$$\Delta_{Fx1} = \frac{F_{\min}\left(N_1\ M - N\ M_1\ \right) - F_{out}\left(\ M - M_1\ \right)}{N\ M_1}$$
$$\Delta_{Fx2} = \frac{F_{\max}\left(N_1\ M - N\ M_1\ \right) - F_{out}\left(\ M - M_1\ \right)}{N\ M_1} \qquad [9]$$

A similar elimination of unknown variables can be performed to map the IF range back to the input, giving the designer insight into what portions of the IF band are subject to spurious corruption by portions of the input band. Begin once again with the relationships in [5] and [6].

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_x + M\ F_{LO} = F_{out} \qquad [5, 6]$$

The input frequency, F_{in}, is defined to be equal to F_{rf1} and F_x equal to Δ_{Fx} + F_{in}:

$$F_{in} = F_{rf1} \qquad F_x = \Delta_{Fx}\Big|_{\Delta F_{\max}} + F_{in}\Big|_{F_{in} = F_{rf1}} = \Delta F_{\max} + F_{rf1}$$

Once again F_{LO} is varying and F_{out} is known, therefore F_{LO} is eliminated from the pair of equations in [5,6].

$$\begin{array}{l} M\ N_1\ F_{rf1} + M\ M_1\ F_{LO} = M\ F_{out} \\ \underline{-\left(M_1\ N\left(\Delta F_{\max} + F_{rf1}\right) + M_1\ M\ F_{LO}\ \right) = -\ M_1\ F_{out}} \\ M\ N_1\ F_{rf1} - M_1\ N\left(\Delta F_{\max} + F_{rf1}\right) = F_{out}\ \left(\ M - M_1\ \right) \end{array} \qquad [10]$$

The IF passband is 2 ΔF_{max} in width, therefore substituting ± ΔF_{max} gives the two frequencies at the input which define the mapping of the IF output passband to the input.

$$F_{rf1} = \frac{M_1\ N\left(-\ \Delta F_{\max}\right) + F_{out}\left(\ M - M_1\ \right)}{M\ N_1 - M_1\ N}$$
$$F_{rf2} = \frac{M_1\ N\left(\ \Delta F_{\max}\right) + F_{out}\left(\ M - M_1\ \right)}{M\ N_1 - M_1\ N} \qquad [11]$$

Case 2: *Fixed Local Oscillator, Swept Input*

We begin with the following pair of equations:

$$N_1\, F_{in} + M_1\, F_{LO} = F_{out}$$
$$N\, F_x + M\, F_{LO} = F_{out} \qquad [5, 6]$$

Setting [5] equal to [6] and solving for F_x gives:

$$N_1\, F_{in} + M_1\, F_{LO} = N\, F_x + M\, F_{LO}$$
$$F_x = \frac{N_1\, F_{in} + (\, M_1 - M\,) F_{LO}}{N} \qquad [12]$$

Using the fact in [4] and substituting F_{min} and F_{max} for F_{in} gives the following:

$$\Delta_{F.x1} = \frac{F_{\min}\,(\, N_1 - N\,) + (\, M_1 - M\,) F_{LO}}{N}$$
$$\Delta_{F_{x2}} = \frac{F_{\max}\,(\, N_1 - N\,) + (\, M_1 - M\,) F_{LO}}{N} \qquad [13,14]$$

A similar elimination of unknown variables can be performed to map the IF range back to the input, giving the designer insight into what portions of the IF band are subject to spurious corruption by portions of the input band. Begin once again with the relationships in [5] and [6].

$$N_1\, F_{in} + M_1\, F_{LO} = F_{out}$$
$$N\, F_x + M\, F_{LO} = F_{out} \qquad [5, 6]$$

The input frequency, F_{in}, is defined to be equal to F_{rf1} and F_x equal to $\Delta_{Fx} + F_{in}$:

$$F_{in} = F_{rf1} \qquad F_x = \Delta_{F_x} + F_{in} = \Delta_{F_x} + F_{rf1}$$
$$= \Delta_{F_x} + F_{in} = -\Delta_{F_x} + F_{rf2}$$

F_{out} is varying, therefore F_{out} is eliminated from the pair of equations in [5,6].

$$N_1\, F_{rf} + M_1\, F_{LO} = N\, F_x + M\, F_{LO}$$
$$F_{rf} = \frac{N\, F_x + (\, M - M_1\,) F_{LO}}{N_1} \qquad [15]$$

The IF passband is 2 ΔF_{max} in width, therefore substituting $\pm\, \Delta F_{max}$ gives the two frequencies at the input after substituting for F_x.

$$F_{rf1} = \frac{\Delta_{F_x}\, N + (\, M - M_1\,) F_{LO}}{N_1 - N}$$
$$F_{rf2} = \frac{-\Delta_{F_x}\, N + (\, M - M_1\,) F_{LO}}{N_1 - N} \qquad [16]$$

Case 3: _Fixed Local Oscillator, Swept Input_

A modification in the expressions used is incorporated here to accomodate what is termed the "transmitter" case. In the transmitter case the input signal at RF is assumed "clean" with the only spurious products generated at the output due to mixer spurious responses, i.e. no contribution from nearby signals at the input. In the receiver cases the potential for nearby signals to generate spurious output mechanisms is the concern, contrasted here with internally generated signals occurring within ΔF MHz of the desired output (transmitted) signal for the transmitter case.

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_{in} + M\ F_{LO} = F_x \qquad [17,18]$$

with F_x defined to be:

$$\Delta F_x = F_x - F_{out} \qquad [19]$$

The quantity fixed in value is the local oscillator frequency, F_{LO}. The derivation proceeds by elimination of the swept input frequencies from both [17] and [18].

$$\begin{array}{l} N\ N_1\ F_{in} + N\ M_1\ F_{LO} = N\ F_{out} \\ -\left(N_1\ N\ F_{in} + N_1\ M\ F_{LO} - N_1\ \Delta F_x = N_1\ F_{out} \right) \\ \hline \left(N\ M_1 - N_1\ M \right) F_{LO} + N_1\ \Delta F_x = \left(N - N_1 \right) F_{out} \end{array} \qquad [20]$$

$$\Delta_{Fx1} = \frac{\left(N_1\ M - N\ M_1 \right) F_{LO} + \left(N - N_1 \right) \left(F_{out} \right)_{\min}}{N_1} \qquad [21]$$

$$\Delta_{Fx1} = \frac{\left(N_1\ M - N\ M_1 \right) F_{LO} + \left(N - N_1 \right) \left(F_{out} \right)_{\max}}{N_1} \qquad [22]$$

The ΔF calculated in equations [21] and [22] is the frequency offset from the output for a given (M,N) spurious mixer product.

In a manner similar to that for Cases 1 and 2 the frequencies at the input causing the spurious products at the output are derived.

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_{in} + M\ F_{LO} = F_x \qquad [23, 24]$$

with F_x defined to be:

$$F_x = \Delta F_{\max} + F_{out} \qquad [25]$$

$$\begin{array}{l} N\ N_1\ F_{in} + N\ M_1\ F_{LO} = N\ F_{out} \\ -\left(N_1\ N\ F_{in} + N_1\ M\ F_{LO} - N_1 \left(\Delta F_x \right)_{\max} = N_1\ F_{out} \right) \\ \hline \left(N\ M_1 - N_1\ M \right) F_{LO} + N_1 \left(\Delta F_x \right)_{\max} = \left(N - N_1 \right) F_{out} \end{array} \qquad [26]$$

$$F_{rf1} = \frac{\left(N_1\ M - N\ M_1 \right) F_{LO} - N_1\ \left(\Delta F_x \right)_{\min}}{N_1 - N} \qquad [27]$$

$$F_{rf2} = \frac{\left(N_1\ M - N\ M_1 \right) F_{LO} - N_1\ \left(\Delta F_x \right)_{\max}}{N_1 - N} \qquad [28]$$

Case 4: <u>*Fixed Input Frequency, Swept Local Oscillator*</u>

In the final case to be presented, the input frequency is fixed while the local oscillator varies. In keeping with the other derivations performed , the swept local oscillator frequency term is eliminated from the pair of equations. Once again, the transmitter case is considered, looking for output spurious frequencies within ΔF MHz of the output frequency.

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_{in} + M\ F_{LO} = F_x \qquad [17,18]$$

with F_x defined to be:

$$\Delta F_x = F_x - F_{out} \qquad [19]$$

$$\begin{array}{l} M\ N_1\ F_{in} + M\ M_1\ F_{LO} = M\ F_{out} \\ -\left(M_1\ N\ F_{in} + M_1\ M\ F_{LO} - M_1\ \Delta F_x = M_1\ F_{out} \right) \\ \hline \left(M\ N_1 - M_1\ N \right) F_{RF} + M_1\ \Delta F_x = \left(M - M_1 \right) F_{out} \end{array} \qquad [29]$$

$$\Delta_{Fx1} = \frac{\left(M - M_1 \right) \left(F_{out} \right)_{\min} - \left(M\ N_1 - M_1\ N \right) F_{in}}{M_1} \qquad [30]$$

$$\Delta_{Fx2} = \frac{\left(M - M_1 \right) \left(F_{out} \right)_{\max} - \left(M\ N_1 - M_1\ N \right) F_{in}}{M_1} \qquad [31]$$

In a manner similar to that for Cases 1 and 2 the frequencies at the input causing the spurious products at the output are derived.

$$N_1\ F_{in} + M_1\ F_{LO} = F_{out}$$
$$N\ F_{in} + M\ F_{LO} = F_x$$
[23, 24]

with F_x defined to be:

$$F_x = \Delta F_{\max} + F_{out} \qquad [25]$$

$$\begin{array}{l} M\ N_1\ F_{in} + M\ M_1\ F_{LO} = M\ F_{out} \\ -\left(M_1\ N\ F_{in} + M_1\ M\ F_{LO} - M_1 \left(\Delta F_x \right)_{\max} = M_1\ F_{out} \right) \\ \hline \left(M\ N_1 - M_1\ N \right) F_{RF} + M_1 \left(\Delta F_x \right)_{\max} = \left(M - M_1 \right) F_{out} \end{array} \qquad [32]$$

$$F_{rf1} = \frac{\left(M\ N_1 - M_1\ N \right) F_{RF} + M_1\ \left(\Delta F_x \right)_{\min}}{M - M_1} \qquad [33]$$

$$F_{rf2} = \frac{\left(M\ N_1 - M_1\ N \right) F_{RF} + M_1\ \left(\Delta F_x \right)_{\max}}{M - M_1} \qquad [34]$$

Specific Numerical Examples

Case 1:

RF Range: 200 - 400 MHz
IF Range 70 MHz
LO Range: 130 - 330 MHz

For an (M,N) spur equal to (4,-3), spurious products appear over the range (-50, 16.666) MHz of the desired output. Input Frequencies within ± 50 MHz of the desired input signal produce spurious products at the IF between frequencies (200, 500) MHz for the same (-4,3) order spur.

$$\Delta F_{x1,x2} = \frac{F_{\min}\left(N_1\ M - N\ M_1 \right) - F_{out}\left(M - M_1 \right)}{N\ M_1} = \frac{200\left(4-3 \right) - 70\left(4+1 \right)}{3} = -50.0$$
$$= \frac{400\left(4-3 \right) - 70\left(4+1 \right)}{3} = -16.6666$$

$$F_{rf1,rf2} = \frac{M_1\ N\left(-\Delta\ F_x \right) + F_{out}\left(M - M_1 \right)}{M\ N_1 - M_1\ N} = \frac{(-1)(-3)\left(-50 \right) + 70\left(4+1 \right)}{(4)(1) - (-1)(-3)} = 200$$
$$= \frac{(-1)(-3)\left(+50 \right) + 70\left(4+1 \right)}{(4)(1) - (-1)(-3)} = 500$$

Double-check the first pair of answers for ΔF calculations:

$$N\left(F_{in} + \Delta F_x \right) + M\ F_{LO} \equiv F_{IF}$$
$$-3\left(200 - 50 \right) + 4\left(130 \right) = 70.0 \qquad YES$$
$$-3\left(200 + 16.6666 \right) + 4\left(330 \right) = 70.0 \quad YES$$

Double-check second pair of answers for input frequencies suffering from spurious (-4, 3) mechanism.

$$-3\left(F_{RF} - 50 \right) + 4\left(F_{RF} - 70 \right) \equiv 70 \quad F_{RF} = 200 \qquad YES$$
$$-3\left(F_{RF} + 50 \right) + 4\left(F_{RF} - 70 \right) \equiv 70 \quad F_{RF} = 500 \qquad YES$$

Case 2:

RF Range: 300 - 400 MHz
IF Range 50 - 150 MHz
LO Range: 250 MHz

For an (M,N) spur equal to (3, -2), spurious products appear over the range (50, -100) MHz of the desired output. Input Frequencies within ± 50 MHz of the desired input signal produce spurious products at the IF between frequencies (366.666, 300.0) MHz for the same (3, -2) order spur.

$$\Delta F_{x1,x2} = \frac{F_{\min}\left(N_1 - N \right) - F_{LO}\left(M_1 - M \right)}{N} = \frac{300\left(1+3 \right) + 250\left(-1-3 \right)}{-2} = +50.0$$

$$= \frac{400\left(3 \right) - 4\left(250 \right)}{-2} = -100.0$$

$$F_{rf1,rf2} = \frac{\Delta F_x \, N + \left(M - M_1 \right) F_{LO}}{N_1 - N} = \frac{-50\left(-2 \right) + \left(3+1 \right) 250}{1+2} = 366.6666$$

$$= \frac{50\left(-2 \right) + \left(3+1 \right) 250}{1+2} = 300.0$$

Double-check the first pair of answers for ΔF calculations:

$$N\left(F_{in} + \Delta F_x \right) + M\,F_{LO} \equiv F_{IF}$$

$$-2\left(300+50 \right) + 3\left(250 \right) = 50.0 \qquad YES$$

$$-2\left(400-100.0 \right) + 3\left(250 \right) = 150.0 \qquad YES$$

Double-check second pair of answers for input frequencies suffering from spurious (-4, 3) mechanism.

$$-2\left(F_{RF} - 50 \right) + 3\left(250.0 \right) \equiv F_{out} = 116.666 \qquad F_{RF} = F_{out} + F_{LO} = 366.6666 \quad YES$$

$$-2\left(F_{RF} + 50 \right) + 3\left(250.0 \right) \equiv F_{out} = 50.0 \qquad F_{RF} = F_{out} + F_{LO} = 300.0 \quad YES$$

Case 3:

RF Range: 300 - 400 MHz
IF Range 200 - 100 MHz
LO Range: 500 MHz

For an (M,N) spur equal to (-2, 3), spurious products appear over the range (100, -300) MHz of the desired output. Input Frequencies within ± 50 MHz of the desired input signal produce spurious products at the IF between frequencies (137.5, 112.5) MHz for the same (-2, 3) order spur.

$$\Delta F_{x1,x2} = \frac{F_{LO}\left(N_1 M - N\,M_1 \right) - F_{out}\left(N - N_1 \right)}{N_1} = \frac{\left(-1\left(-2 \right) - 3\left(1 \right) \right) 500 + \left(3+1 \right) 100}{-1} = 100.0$$

$$= \frac{\left(2-3 \right) 500 + \left(4 \right) 200}{-1} = -300.0$$

$$F_{rf1,rf2} = \frac{(N_1\ M - N\ M_1)F_{LO} - N_1(\Delta F_x)}{N_1 - N} = \frac{(-1(-2)-3)500.0 + (-50.0)}{-1-3} = 137.5$$

$$= \frac{(2-3)500 + 50.0}{-1-3} = 112.5$$

Double-check the first pair of answers for ΔF calculations:

$$N\ F_{in} + M\ F_{LO} \equiv F_{out} + \Delta F_x$$

$$3(300) - 2(500) = 200 + (-300) \quad YES$$

$$3(400) - 2(500) = 100 + (100) \quad YES$$

Double-check second pair of answers for input frequencies suffering from spurious (-4, 3) mechanism.

$$3(F_{RF}) - 2(500.0) \equiv F_{out} + \Delta F_x = -50 + F_{RF} + 500 \quad F_{RF} = 362.5,\ \ F_{out} = 500 - 362.5 = 137.5 \quad YES$$

$$3(F_{RF}) - 2(500.0) \equiv F_{out} + \Delta F_x = +50 + F_{RF} + 500 \quad F_{RF} = 387.5,\ \ F_{out} = 500 - 387.5 = 112.5 \quad YES$$

Case 4:

RF Range: 100 MHz
IF Range 300 - 500 MHz
LO Range: 200 - 400 MHz

For an (M,N) spur equal to (4, -6), spurious products appear over the range (-100, 500) MHz of the desired output. Input Frequencies within ± 50 MHz of the desired input signal produce spurious products at the IF between frequencies (316.66, 350) MHz for the same (4, -6) order spur.

$$\Delta F_{x1,x2} = \frac{(M - M_1)F_{out} - (M\ N_1 - M_1\ N)F_{RF}}{N_1} = \frac{(4-1)300 - (4+6)100}{1.0} = -100.0$$

$$= \frac{(4-1)500 - (4+6)100}{1.0} = 500.0$$

$$F_{rf1,rf2} = \frac{(N_1\ M - N\ M_1)F_{RF} - M_1(\Delta F_x)}{M - M_1} = \frac{(4+6)100.0 + (-50.0)}{4-1} = 316.666$$

$$= \frac{(4+6)100.0 + (+50.0)}{4-1} = 350.0$$

Double-check the first pair of answers for ΔF calculations:

$$N\ F_{in} + M\ F_{LO} \equiv F_{out} + \Delta F_x$$

$$3(300) - 2(500) = 200 + (-300) \quad YES$$

$$3(400) - 2(500) = 100 + (100) \quad YES$$

Double-check second pair of answers for input frequencies suffering from spurious (-4, 3) mechanism.

$$-6(F_{RF}) + 4(F_{LO}) \equiv F_{out} + \Delta F_x = -50 + F_{LO} + 100 \quad F_{LO} = 216.66,\ \ F_{out} = 100 + 216.66 = 316.666 \quad YES$$

$$-6(F_{RF}) + 4(F_{LO}) \equiv F_{out} + \Delta F_x = +50 + F_{LO} + 100 \quad F_{LO} = 250.0,\ \ F_{out} = 100 + 250.0 = 350.0 \quad YES$$