

$$\cosh(\alpha + j\beta) = \cosh(\alpha)\cos(\beta) + j\sinh(\alpha)\sin(\beta)$$

$$\sinh(\alpha + j\beta) = \sinh(\alpha)\cos(\beta) + j\cosh(\alpha)\sin(\beta)$$

$$\tanh(\alpha + j\beta) = \frac{\sinh(2\alpha) + j\sin(2\beta)}{2\cosh^2(\alpha)\cos^2(\beta) + \sinh^2(\alpha)\sin^2(\beta)}$$

$$\cos(\alpha + j\beta) = \cosh(\beta)\cos(\alpha) - j\sinh(\beta)\sin(\alpha)$$

$$\sin(\alpha + j\beta) = \cosh(\beta)\sin(\alpha) + j\sinh(\beta)\cos(\alpha)$$

$$\text{Power dB} = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

If powers are measured at the same impedance level, then the dB ratio of voltages and currents becomes

$$\text{Voltage dB} = 20\log_{10}\left(\frac{V_1}{V_2}\right) \quad \text{Current dB} = 20\log_{10}\left(\frac{I_1}{I_2}\right)$$

One should not attempt to convert dB voltage and current ratios to power ratios without knowledge of the impedances across which they are measured.

$$\text{Attenuation in nepers: } N = \ln\left(\frac{V_1}{V_2}\right)$$

$$\text{In terms of the ratio of input power to output power at the same impedance level: } N = \frac{1}{2}\ln\left(\frac{P_1}{P_2}\right)$$

$$\alpha = \ln\left(\frac{V_1}{V_2}\right) \quad e^\alpha = \frac{V_1}{V_2}$$

$$20\log_{10}\left(\frac{V_1}{V_2}\right) = 20\log_{10}(e^\alpha) \quad \text{dB} = 20\alpha\log_{10}(e)$$

α is the attenuation in nepers

$$\text{dB} = 8.686\alpha$$



$$S_{11}' = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\text{Power gain: } P = 10\log_{10}\left(|S_{21}|^2\right) \text{ and } |S_{21}| = |S_{12}| \quad \text{so: } |S_{21}| = |S_{12}| = 10^{P/20}$$

Example:

A pad of 6 dB. Output VSWR to match is 1.65

$$\Gamma_L = \frac{VSWR - 1}{VSWR + 1} = \frac{1.65 - 1}{1.65 + 1} = 0.24528 \Rightarrow 20\log_{10}(0.24528) = -12.21\text{dB}$$

For the 6 dB pad, $|S_{21}| = |S_{12}| = 10^{-6/20} = 0.501187$

Inserting into $S_{11}' = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

$$S_{11}' = S_{11} + \frac{(0.501187)^2 0.24528}{1 - 0} = 0.061612$$

Now translate this S_{11} term into VSWR:

$$VSWR = \frac{1 + |S_{11}|}{1 - |S_{11}|} = \frac{1 + 0.061612}{1 - 0.061612} = 1.1313$$

$$\text{Return Loss} = 20\log_{10}\left(\frac{VSWR - 1}{VSWR + 1}\right) = -24.2\text{ dB}$$

Check: With the pad inserted the return loss should equal the original return loss at VSWR = 1.65:1, improved by twice the loss of the pad (2 * 6 dB).

Original return loss of 12.21 dB + 2 x 6 dB equals 24.21 dB return loss. Check