Definitions of Power and
Related Methods of Calculation
by Jeff Crawford

The determination of power in a circuit is considered by most to be a straight forward mathematical operation involving the product of voltage and current. Is this always true? What other factors have to be considered before quickly applying this rule of thumb to any and all situations? Where does a factor of "2" come into the simple relationships sometimes used for power? What is the difference between average power and instantaneous power? Follow the ensuing paragraphs for answers to these and related questions.

Calculation of DC Power

The determination of power is most easily handled in DC circuits where no time-varying considerations are necessary. In a DC circuit the instantaneous and average power are equal and are given by **V×I**. The instantaneous power is the power in a component or circuit at a specific instant in time. The average power is the power, that if measured over a significant period of time and averaged, i.e. over seconds, is found in a component or circuit. By definition, the current and voltage in DC circuits are not varying with time; therefore instantaneous power and average power are equivalent terms. In simple DC circuits we seldom encounter inductors or capacitors; these two elements introduce complications into AC power calculations as we will see shortly.

Calculation of AC Power

Under conditions of sinusoidal excitation the governing relationships used in DC circuits must be modified. Consider, for example, the voltage and current in an AC circuit given by:

$$v(t) = V_m \cos(\omega t + \alpha) \qquad i(t) = I_m \cos(\omega t + \alpha - \theta) \qquad [\,1, 2\,]$$

"α" is some arbitrary phase angle associated with both the voltage and current (to account for conditions at t = 0), while θ is a phase difference between the voltage and current, respectively. This phase difference is generally attributed to capacitive or inductive effects. With time as a reference, capacitors retard voltage compared to current while inductors retard current compared to voltage. V_m and I_m refer to the maximum magnitude the voltage and current may achieve at any instant in time. A phase difference between two different voltages is shown in Figure 1, where the voltage V_2 leads V_1 in phase by 60 degrees. It is equally correct to say that V_1 lags V_2 in phase by 60 degrees. The period for each sine wave is one second.

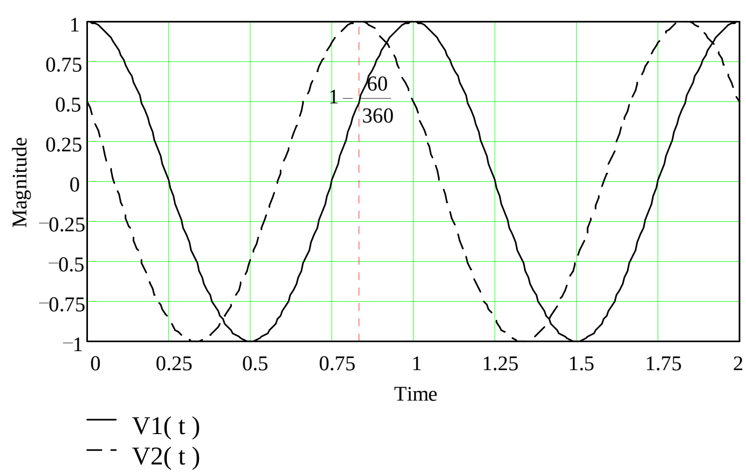

Figure 1
Two Voltage Waveforms: V_2 Leading V_1 in Phase by 60 Degrees

Using the expressions developed in [1] and [2] and performing multiplication, the instantaneous power at each time "t" is given by the following:

$$p(t) = v(t)\,i(t) = V_m\,I_m \cos(\omega t + \alpha)\cos(\omega t + \alpha - \theta) \qquad [\,3\,]$$

Applying the trigonometric identity in [4] to [3] gives us the intermediate result we are seeking.

$$\cos(\gamma)\cos(\beta) = \frac{1}{2}\left[\cos(\gamma - \beta) + \cos(\gamma + \beta)\right] \qquad [\ 4\ ]$$

$$p(t)\Big|_{\alpha=0} = V_m I_m \cos(\omega t)\cos(\omega t - \theta) = \frac{V_m I_m}{2}\left[\cos(\theta) + \cos(2\omega t - \theta)\right] \qquad [\ 5\ ]$$

$$\boxed{p(t) = \frac{V_m I_m}{2}\left[\cos(\theta) + \cos(2\omega t - \theta)\right]} \qquad [\ 5\ ]$$

The expression in [5] is the instantaneous power for a system energized by a sinusoidal voltage. The following facts related to the instantaneous power can be implied from this equation: 1) the first part of the instantaneous power is a constant and depends only on the phase difference θ between the voltage and current, and 2) the second part is time-varying at a frequency double that of the input voltage signal.

As discussed earlier, the average power is the power we would find in a system if measured over an extended period of time. Upon further review of [5] it is seen that the part of the expression which is time-varying (the second part) will, over time, average to zero. The average of all sine waves is zero by definition, unless the sine wave is accompanied by a DC offset voltage. Therefore, the average power for a system energized by sinusoidal inputs is

$$P_{avg} = AVG\left[p(t)\right] = \frac{V_m I_m}{2}\cos(\theta) \qquad [\ 6\ ]$$

From [6] it can be implied that as the phase difference between the voltage and current changes (as a function of differing amounts of capacitive or inductive effects in the system), the average power will also fluctuate. In electric power utility operations large users of electricity have additional levies placed on their power usage rates if the phase angle between the voltage and current exceeds more than perhaps 10 degrees in phase. The total power that the consumer needs to have delivered to his site is called apparent power [7], while the power actually used by the consumer is the average power already given by [6]. The difference between these two terms is that apparent power includes power required by capacitive or inductive effects of a particular installation, while average power considers only the power delivered to a pure resistance. From circuit theory we know that power delivered to a capacitor or inductor during one part of the AC cycle is returned back to the remainder of the circuit through the rest of the AC cycle. Therefore inductors and capacitors use no net power over the course of a cycle. The phase angle θ for an average home will be very close to zero, however a large industrial user employing large electric motors or other equipment, which is primarily inductive, will have a phase angle of perhaps 10 to 30 degrees. The electric utility company must operate within the following bounds:

$$P_{app} = \frac{V_m I_m}{2} \qquad [\ 7\ ]$$

Power usage charged to customer: $P_{avg} = \frac{V_m I_m}{2}\cos(\theta)$

Power that must be delivered to customer: $P_{app} = \frac{V_m I_m}{2}$

Consider the case of an industrial user whose loading conditions on the utility lines cause a phase angle between the voltage and current of 25 degrees. For every watt of power which is delivered to the customer, the utility can charge for only cos(25) or 90.6% of the power delivered. The reason phase angle and power factor, to be defined shortly, are watched so carefully should be *apparent*! As the phase angle gets larger, the electric utility must deliver more and more power - increased size of transmission and power generation structure - while the customer consumes less and less real power - the power the electric company can legally charge them for. In actuality, utility companies use large banks of capacitors placed strategically through their system to help mitigate the inductive effects and improve power transmission efficiency.

$$\text{Power Factor} = \text{PF} = \frac{average\ power}{apparent\ power} = \frac{2\,P}{V_m I_m} \qquad [\ 8\ ]$$

Effective Values of Voltage and Current

Thus far in this article, peak values of voltage and current have been used. It is commonplace to use what are termed *rms* values of voltage or current. The rms value of a current, sometimes called equivalently the *effective value* of current, is equal to the value of DC current which, flowing through a resistance R, delivers the same power to R as a periodic current. The rms current is the square root of the integral - i.e. summation - of current squared over one period as expressed in [9].

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \; dt} \qquad [\ 9\ ]$$

When the mathematical operation in [9] is carried out for <u>strictly sinusoidal waveforms</u>, it is found that a factor of the square root of 2 relates peak and rms values.

$$I_{rms} = \frac{I_m}{\sqrt{2}} \qquad V_{rms} = \frac{V_m}{\sqrt{2}} \qquad [\ 10\ ]$$

The average power expressed in [6] can be written equivalently in rms values as shown in [11].

$$P_{avg} = AVG\left[\ p(\ t\ )\ \right] = \frac{V_m\, I_m}{2}\cos(\ \theta\ ) = V_{rms}\, I_{rms}\cos(\ \theta\ ) \qquad [\ 11\ ]$$

Conditions Maximizing Transfer of Power

The accompanying diagram in Figure 2 illustrates a voltage source V_{in}, and its associated internal resistance R_1, driving a load with characteristic resistance of R_2. The fundamental question we need to answer is what conditions with respect to the values of R_1 and R_2 , maximize the amount of power transferred to the load resistor R_2? For those readers familiar with calculus, we can write an expression for the power transferred to the load, take the derivative of this expression with respect to the load R_2, set this equal to zero and identify what value of R_2 that gives maximum transfer. For those readers not familiar with this technique, we will use the same expression for power transferred to the load and evaluate subsequently this for a range of load resistance R_2.

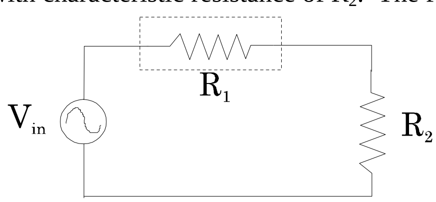

Figure 2
Input Signal Source, Generator Resistance R_1, and Load R_2

$$P_{load} = \frac{V_{R_2}^{\ 2}}{R_2} = \frac{V_{in}^{\ 2}\, R_2}{(\ R_1 + R_2\ )^2} \qquad [\ 13\ ]$$

With an inherent generator resistance of 1 kΩ the maximum amount of power (relative) delivered to the load occurs when the source and load resistances are equal. When the load resistance is less than the source, more power is dissipated in the generator impedance than in the load, and when the load resistance is greater than the source impedance, less than optimum power is dissipated in the load.

Maximum power is transferred to a load when the source and load impedances are equal.

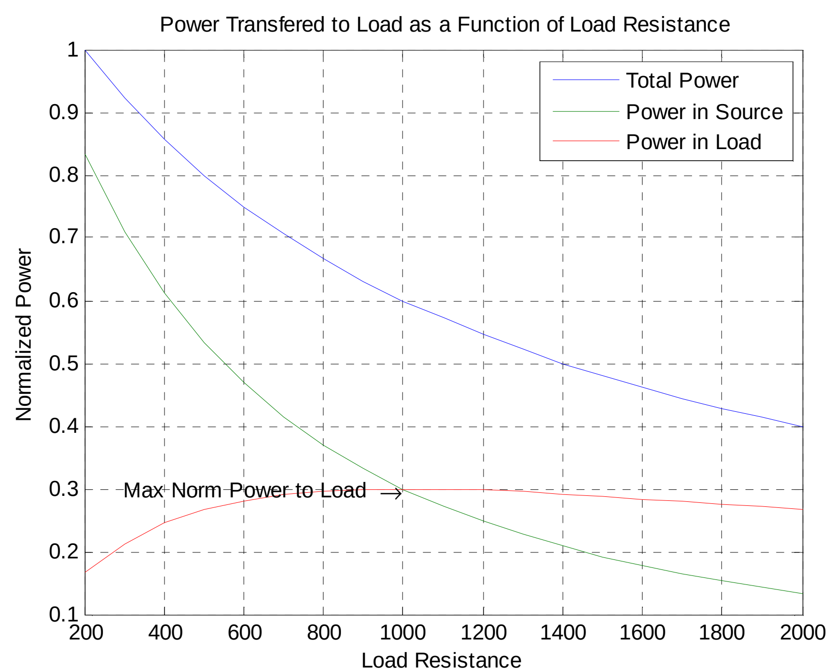

Figure 3
Power Dissipated in Source and Load Resistances for Various Load Resistance Values; Source Resistance Constant at 1 kΩ

In Figure 3 one can see that while the "power in the circuit" is greatest for lower load resistances, such as R_L = 400 ohms, the percentage of power that is consumed by the load resistor compared to that consumed by the source is less than optimum, "optimum being 50%. At R_L of 1000 ohms, 0.3 normalized power is absorbed by the load R_L and this is 50% of the total normalized power in the circuit, 0.6.

In a subsequent treatment on power we will look more in depth at power in circuits containing inductive and capacitive elements, and how their presence impacts peak voltages and currents over time. We will also look at actual expressions for the voltage and current in a simple R-L or R-L-C circuit and see the impacts of apparent power and average power in an actual circuit.

Additional Notes

Starting with [9] above:

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \ dt} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t)\, dt} = \sqrt{\frac{I_m^2}{2T}\int_0^T \frac{1+\cos(2\omega t)}{2} dt}$$

$$= \sqrt{\frac{I_m^2}{T}\left[t + \frac{\sin(2\omega t)}{2\omega}\right]_0^T} = \sqrt{\frac{I_m^2 T}{2T}} = \frac{I_m}{\sqrt{2}}$$