Introduction

Transmission Line Transformers (TLT) are an oddity, in my opinion. They are depicted schematically as coupled inductors and yet the reader is told by most authors that they are to be considered transmission lines. In fact, for an N-turn configuration, if the coupling coefficient is made to be close to 1.0 for all combinations of inductors, you will not get the anticipated results. The coupling must be restricted to pairs of coils (in most cases; depends on physical construction). There are exceptions you will see, but much later in this tutorial.

In dealing with TLTs one must always remember essentially a) conservation of current and b) conservation of voltage. In other words, for short "transmission lines" as we use here, the current at the output is almost the same as that at the input and the same is true for the voltage. So while meaningful results can be gathered from a SPICE simulation, one must be careful to keep in mind, "transmission line mode", "transmission line mode".

General Analysis



deal with the low frequency end.

Figure 1-2 to the left is directly out of Transmission Line Transformers¹ by Jerry Sevick. This figure is found in Ruthroff's original paper as well.

As with conventional transformers, Ruthroff divided his analysis into two parts: 1) high frequency and 2) low frequency. For the highfrequency part we assume that the reactance of the coiled lines occurs at such a degree as to isolate the input and output circuits, leaving only transmission line currents and voltages. In fact, two of the equations to be presented are exactly that: transmission line equations. The second pair

Low-Frequency End

$$V_g = (I_1 + I_2)R_g - V_2 + I_2R_L$$
 (1,2)

These loop equations are easily developed by writing the input and output voltage loop equations found in Figure 1-2. The next pair of equations are the conventional transmission line equations for current and voltage along a line of length l.

$$V_1 = V_2 \cos(\beta l) + jI_2 Z_o \sin(\beta l) \qquad I_1 = I_2 \cos(\beta l) + j\frac{V_2}{Z_o} \sin(\beta l) \quad (3,4)$$

This set of equations is solved for the output power P_o, where $P_o = \left| I_2 \right|^2 R_L$ (5)

 $V_{g} = (I_{1} + I_{2})R_{g} + V_{1}$

Equation (3) can be substituted into (1) to eliminate² V_1 as a variable and in so doing, the following three simultaneous equations can be written.

Cramer's Rule can then be used to solve this set of simultaneous equations in (6).

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$$\begin{bmatrix} V_{g} \\ V_{g} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{g} & R_{g} + jZ_{o}\sin(\beta l) & \cos(\beta l) \\ R_{g} & R_{g} + R_{L} & -1 \\ -1 & \cos(\beta l) & \frac{j}{Z_{o}}\sin(\beta l) \end{bmatrix} \begin{bmatrix} I_{1} \\ I_{2} \\ V_{2} \end{bmatrix}$$
(6)
$$I_{2} = \frac{\begin{vmatrix} R_{g} & V_{g} & \cos(\beta l) \\ R_{g} & V_{g} & -1 \\ -1 & 0 & \frac{1}{Z_{o}}\sin(\beta l) \\ \det(A) \end{vmatrix} = \frac{V_{g}(1 + \cos(\beta l)) + V_{g}(1 + \cos(\beta l))}{2R_{g}(1 + \cos(\beta l)) + R_{L}\cos(\beta l) + j(R_{g}R_{L} + Z_{o}^{2})\sin(\beta l)/Z_{o}}$$
(7)

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There is a maximizing Z_o which if used, improves the power out P_o .

$$Z_o = \sqrt{R_g R_L} \quad (8)$$

For this value of Z_0 the power output is:

$$P_{o} = |I_{2}|^{2} R_{L} = \frac{V_{g}^{2} (1 + \cos(\beta l))^{2} R_{L}}{\left[2R_{g} (1 + \cos(\beta l)) + R_{L} \cos(\beta l)\right]^{2} + 4R_{g}R_{L} \sin^{2}(\beta l)}$$
(9)

There is a load R_L, which maximizes power delivery as well and it is determined by evaluating $dP_a/dR_L = 0$

$$R_{L} = \frac{2R_{g}\left(1 + \cos\left(\beta l\right)\right)}{\cos\left(\beta l\right)} \quad (10)$$

The ratio of power output to available power is

$$\frac{P_o}{P_A} = \frac{\left(1 + \cos\left(\beta l\right)\right)^2}{\frac{5}{4} \left(1 + \frac{6}{5} \cos\left(\beta l\right) + \cos^2\left(\beta l\right)\right)} \quad (11)$$

When $\cos(\beta l)$ becomes significantly less than 1, the frequency response of this circuit begins to roll off and the power gain is zero when $\beta l = \pi$.

Simetrix Simulations and Low-Frequency Assessments

The following pages outline several Ruthroff implementations, focusing on SPICE-like simulations to assess their performance (remembering the rules about coupling coefficient pairs) and performing some associated hand calculations for verification and further understanding.



Conservation of current tells us that i_1 (current in L_1) must be the same as i_2 (current in L_2). The green trace to the right is the current entering L_1 . It is positive because current entering devices is "+" and here the current is entering L_1 , L1-P.

The red curve is the current exiting L_2 ; shown as negative because the current is exiting, L2-P.



Figure 2 Voltage at Dot of L_1 and L_2

The magnetic core ensures that current entering is the same as that exiting as already stated. This is also a principle of transmission lines in general.

The phasing dots are such that current i_1 enters L_1 's dot while the current exiting L_2 does so at its dot. For the mutual inductance, M, to be positive, i_1 and i_2 must both enter their respective dots or leave their respective dots; we have a mix of i_1 entering its dot while i_2 leaves its dot, therefore M is negative.



Figure 3 TLT - Probes on Opposite Ends

Retaining the phasing of Figure 1 but moving the probes as in Figure 3, the current is exiting L_1 , L1-N, so it is negative at the outset while the current entering L_2 , L2-N, is positive at the outset.

It is still true as above, that current enters the dot of L_1 and due to the magnetics an equal current exits at the dot of L_2 . We can look at the current in the two legs solely as a transmission line¹ and recognize the



Figure 4 Probe Output at Ends of L1 and L2

¹ Radio Frequency Design, Hayward, Wes, The American Radio Relay League, 1994, pg. 147

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currents must be equal and opposite.

The mutual inductance between the coils is governed by the coefficient of coupling, here equal to 0.99. Therefore, if $L_1 = L_2 = 0.01$, M_{12} and $M_{21} = 0.99 \times 0.01 = 0.0099$. In true TLTs this "coupling" is not present; the phasing dots are used to take into consideration the manner in which the transformer is physically constructed.

Because the current exits L_2 at its dot while it enters L_1 at its dot, the sign of the mutual inductance term is negative as already explained. M is positive if i_1 and i_2 each enter the dotted terminal or each enters the undotted terminal. When one is one way and the other the opposite way, the fluxes are in opposition so M will be negative.

The hand calculation performed below results in the same steady-state magnitude of current shown in Figure 1. The use of Laplace Transforms results in the following equations.

$$1 = 50i(s) + sL_1i(s) - sMi(s) + 50i(s) + sL_2i(s) - sMi(s)$$

$$1 = 100i + 2si(L - M)$$

 $1 = i(s) [100 + j2\pi 2f(0.01 - 0.0099)]$

$$|i| = \frac{1}{\sqrt{100^2 + 1256.6^2}} = 0.793 \, ma$$



Figure 5 Summarizing

Essentially the element dictating the amount of current flowing is the amount of resistance in the circuit. As the coupling factor K_{12} gets closer and closer to 1, the flux linkages of L_1 are canceled by the flux linkages of L_2 thus negating the inductance effects leaving only the

K ₁₂ Link Coupling	Current, mA
0.99	0.7983
0.995	1.56
0.999	6.196
0.9999	9.883
1.0	10





resistance. In fact, when K_{12} = 1.0 the current attained is 10 mA, what you'd expect with 100 Ω in the circuit.

The phasing of the windings in these first examples is such that the mutual inductance is negative, therefore a high percentage of the flux from one coil, L_1 , is negated by the second coil, L_2 . Because of

this, smaller magnetics than perhaps first envisioned can be used since there is minimal resultant flux in the core itself.

Now what happens if the phasing on L₂ is flipped?



Figure 7 TLT With Phasing Changed

When the phasing of L_1 and L_2 is reversed, the voltage across R_3 cancels out entirely. Current entering L_1 's dot hits the top part of R_3 and an equal amount of current, albeit in the opposite direction, exits L_2 at its dotted side and is on the other side of R_3 . The voltage differential across R_3 is theoretically zero for perfect coupling between L_1 and L_2 . In this case M is positive (current enters both dots). For positive M the net





Figure 9 Voltage Across R₃ With Phase Change



Figure 8 Probe Output for Phasing Change

Shown to the left is the close-to-zero voltage across R_3 , thus literally no current flows through R_3 . As the coupling coefficient approaches 1.0, the smaller the voltage differential will be across R_3 .

We calculated 3.9789 μA of current through R_3 so the voltage across R_3 should be 199 μV which, with a differential probe, is shown to be exactly that.

When the phasing was changed, the fluxes are additive so the current sees not only the self-inductance of L₁ and L₂ but also the effects of the mutual inductance which is close to 2 x L₁. $1 = 50i(s) + sL_1i(s) + sMi(s) + 50i(s) + sL_2i(s) + sMi(s)$ 1 = 100i + 2si(L + M) $1 = i(s) [100 + j2\pi 2f (0.01 + 0.0099)]$ $|i| = \frac{1}{\sqrt{100^2 + 250,070.775^2}} = 3.9989 \ \mu A$

This is the current used to calculate the \sim 199 μV appearing across $R_3.$

4:1 Step-Up and Step Down TLTs



Figure 10 is an ideal 4:1 TLT, transforming 50 ohms down to 12.5 theoretically. Here current enters L_1 's dot while current exits L_2 's dot, thus the mutual inductance is negative, largely canceling out the fluxes from L_1 and L_2 individually, BUT also isolating the output from the input.

Figure 11 shows the results of the 4:1 TLT. The output current, shown in green, is twice as much as the input current, shown in red. Conservation of energy allows us to write an expression, $P_{in} = P_{out}$.

$$i_{1}^{2} R_{g} = i_{2}^{2} R_{L} \qquad \frac{i_{2}}{i_{1}} = \frac{20 \ mA}{10 \ mA} = 2 \qquad so \quad i_{2} = 2i_{1}$$
$$i_{1}^{2} R_{g} = i_{2}^{2} R_{L} = (2i_{1})^{2} R_{L}$$
$$i_{1}^{2} R_{g} = 4i_{1}^{2} R_{L} \qquad so \quad R_{g} = 4R_{L}$$

R_g of 50 Ω is four-times that of the load, R_L = 12.5 $\Omega.$



Figure 11 Current at Input and Output of 4:1 TLT



Figure 12 Current Flow in 4:1 TLT

In attempting to understand the TLT, it is of paramount importance to always remember that for one pair of coupled inductors representing the TLT, current flows one direction in one of the inductors while it <u>must</u> flow the opposite direction in the other inductor. When current enters one dot and flows out the other dot, this is the case for negative mutual inductance when representing mutually coupled inductors, which is what we want here. The coupled pair of coils is to emulate a power cord, for example, one wire bringing current to a light bulb and the other allowing the current to flow back to the source, completing the circuit.

In Figure 12 we see at the node to the right of L_1 , i_1 is entering from L_1 and i_2 is entering, having come through L_2 . The result is $i_1 + i_2$ or 2i in the load R_2 . Conservation of energy says if one has twice as much

current the load impedance is 1/4th that of the driving source. SO, more current at the output means lower impedance at the load.

Contrast this 4:1 TLT case with a 1:4 case.



Figure 13 is an example of a 1:4 TLT. Note that in the aforementioned 4:1 case, current summation occurred just before the load, in effect doubling the current in the load which lead to the 4:1 transformation ratio. In this case the current branching occurs next to the source, opposite the load, and in this case the current coming from the "source end" of the TLT is twice as high.

The results from the Simetrix simulation are shown in Figure 14 and verify that indeed, twice as much current is flowing at the input compared to that in the output.

Consider Figure 15. It is easily seen that the current i_1 flows from the source through L_1 to ground; this is left-to-right. So the current in L_2 must, as discussed before, flow from right-to-left (when the phasing dots are as shown). Assigning these properties in Figure 15 reveals that a current i_1 and a current i_2 must emanate from the source V_1 , with i_1 going through L_1 and i_2 going through L_2 . Since $i_1 =$ i_2 , twice as much current flows at the input compared to only i_2 at the output, thus a 1:4 TLT.



Figure 15 Current Directions in 1:4 TLT



Figure 14 1:4 TLT Results

Let us now consider a more complex scenario³ involving two cables, i.e. two pairs of coupled inductors. In the case to be presented now in Figure 16, all the current summation is being performed near the load so we can safely assume that the load impedance is lower than the source impedance. In this case, as will be seen shortly, this is a 9:1 impedance step-down TLT (9 $R_L = R_g$). This impedance step-down implies that the load current is 3X that appearing at the input. The current at the input is, per the Simetrix simulation, $i_{in} = 9.923$ mA while that at the output, $i_{out} = 29.172$ mA. In the simulation the coupling coefficients used were $K_{12} = 0.995$ between L_1 and L_2 while $K_{34} = 0.995$ between L_3 and L_4 .

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Figure 16 9:1 Ruthroff TLT

Below, Figure 16 is augmented with the current directions and low-frequency current analysis verifying the 3:1 current difference.



Figure 18 9:1 Impedance Transformer

Important Distinctions

Thus far all inductors have occurred in pairs, each with a high coefficient of coupling, with no coupling between one pair of inductors and another pair (if multiple pairs are present). So far each "pair of inductors" has represented a transmission line and as such, no coupling occurs between the other transmission lines. Imagine, however, a case where the "transmission line" is a pair of insulated wires wound on a toroid; the degree of coupling is close to K = 1.0. Consider the case where this pair of insulated wires is joined by a third wire or perhaps a second pair of insulated wires. In such a physical representation ALL the wires share an almost equal degree of coupling. For such cases our original thinking of "pairs of coupled inductors" must be modified to incorporate the additional coupling that is present.

Figure 19 shows a 4:1 TLT using a coaxial cable as the "transmission line". As we have seen earlier, this physical realization can be properly modeled as a pair of coupled inductors. The left TLT in Figure 20 can



Figure 17 Current Appearing in 9:1 Ruthroff

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be modeled as a pair of coupled inductors, however the right TLT in Figure 20 shares mutual coupling between all four wires composing the transmission lines; a different perspective is required. The left TLT in Figure 20 has a pair of insulated wires that schematically/simulation can be represented as two tightly coupled inductors.





Figure 20 2-Wire and 4-Wire TLTs

Figure 19 4:1 TLT Using Coaxial Line

Each inductor and its associated reactance serve to isolate the output from the input circuit while allowing energy to be transmitted to the output circuit by the normal transmission line mode. As an example, however, Figure 13 shows a realization which does not have an inductance between the input and each end of the load while Figure 3 does have the isolation impedance between the input and each side of the load, thus illustrating the importance of reviewing each instantiation. In those cases where an isolation impedance is present between the input and each side of the load, thus forming a balanced output. Transmission properties are inherently broadband if the characteristic impedance of the line is equal to the terminating impedance. If not, standing waves will exist and a dip in the frequency response will occur when the transmission line is a quarterwavelength long.



Figure 21 4:1 Balun

The figure to the left, labeled Fig 1-2, was shown at the beginning of this paper. The labeling of V₁ should be very apparent to the reader, but that of V₂ may be more perplexing. Due to "conservation of current" in the transmission line, "current-in" must equal "current-out" so $I_1 = I_2$ and the fluxes from the two currents cancel in the core. The amount of inductance in each coil is sufficient to isolate the output from the input such that only transmission line currents flow. The passage of current through the top inductor causes a voltage gradient along the length of the inductor, which in turn results in a voltage V₂ that appears across

the top inductor. Therefore, at the output we have, in series, the input voltage V_1 with the additional voltage gradient of V_2 .

In further consideration of Fig 1-2, if we have a voltage $V_{3,4}$ developed across the top inductor, it is reasonable to assume that the voltage across the bottom inductor, $V_{1,2}$ is also equal to $V_{3,4}$. Therefore, we can state $V_{1,2} = V_{3,4} = V_1$.

Conservation of power allows us to write $P_{in} = P_{out}$. $I_{in}^{2} R_{in} = I_{o}^{2} R_{out} \quad I_{in} = I_{1} + I_{2} = 2I$ $(2I)^{2} R_{in} = I^{2} R_{out} \quad 4R_{in} = R_{out}$

The figure labeled Fig 6-5 consists of three coupled windings, each with a high degree of mutual coupling. Current I₂ goes through the top inductor while both I₁'s go through the bottom two inductors. The bottom two inductors are in series while the top inductor is by itself. The top inductor sees a current I₂ flowing while a current I₁ flows through both the bottom two inductors, however these are in series so you would expect half the current in each of these compared to the top inductor (2X as much inductance). Therefore, I₂ = 2 I₁.







Fig 4-9—A pictorial view and schematic diagram of a 4:1 three-wire transformer with its outer conductors in parallel.

Case Studies – Different TLT Realizations

1:4 Guanella TLT



Figure 22 Guanella TLT

The inputs are in parallel while the outputs are in series.

Where to begin in assessing the operation of this circuit? A current "i" passes through R₂ and L₁ so an equal current, in the opposite direction, flows through L_2 . Because of this, i also flows through L_3 in the direction indicated, and for a transmission line a current "i" flows through L4 in the opposite direction

as indicated. Note: L_1 and L_2 are tightly coupled as are L_3 and L_4 , however there is no coupling between L_{1}, L_{2} and L_{3}, L_{4} . This "continuity of current" results from the properties of short transmission lines.



Looking at the figure to the left wherein the currents are indicated, we can look at Pin and Pout to determine the impedance transformation

$$P_{in} = P_{out}$$
$$(2i)^2 R = i^2 R_L \qquad 4R = R_L$$

Figure 23 Current Distribution

Let's now consider the voltages within this circuit. Half of the input voltage, $rac{1}{2}ig(2V_{_{in}}ig)$ is dropped

across R₁, leaving the other half across the Guanella TLT and its load. Imbedded in our assessment are the following assumptions: 1) R_{in} looking into the Guanella is equal to R_1 so the system is "matched". Under these conditions half the input voltage is dropped across R₁ while the other half across "the load", 2) the characteristic impedance Z_o of each pair of transmission lines is 100 Ω since they are in parallel and we need to have a net Z_0 of 50 Ω .

$$-2V + V + 2V_{\Delta} = 0$$

 V_{Λ} = voltage across each L

Writing a loop equation: V₁, R₁, L₃, L₂ gives: $V_{\Delta} = -\frac{1}{2}V$ So the voltage across L₃, from left-to-right, is -0.5V while due to the transmission line effects, the voltage on L₄ from left-to-right is also -0.5 V. This voltage and its polarity is counterintuitive.

Write another loop equation from V₁, R₁, across L₁, R₂, across L₄ to obtain:

$$-2V + V - \frac{V}{2} + V_L - \frac{V}{2} = 0 \qquad V_L = 2V$$

Because the inputs are in parallel, the voltage between the left hand sides of L₁ and L₂ is V and similarly between the left hand sides of L₃ and L₄. Because of short transmission line properties, the same voltages occur on the right hand sides of L_1 - L_2 and L_3 - L_4 .



Putting all this together results in the following assessment:

Figure 24 4:1 Guanella Balun

4:1 Ruthroff



Figure 25 4:1 Ruthroff

Use conservation of power to determine the impedance transformation. Current in source = i while current in load = i + i = 2i .

$$\frac{P_{in}}{P_{out}} = 1 = \frac{i^2 R}{(2i)^2 R_I} \qquad R_L = \frac{1}{4}R$$

Now determine the voltage across R_L.

$$\frac{V^2}{R} = \frac{V_L^2}{\left(1/4R\right)} \qquad V^2 = 4V_L^2$$
$$V_L = \frac{1}{2}V$$

Now, what is the voltage across L_1 or L_2 , $V_{A,C}$ or $V_{B,D}$?

$$-2V + V + V_{\Delta} + V_{\Delta} = 0 \qquad \qquad V_{\Delta} = \frac{1}{2}V$$

Due to the transmission line, by definition V_{A,C} is the same as Vb,d.

And finally, what is the voltage $V_{A,B}$ or, for that matter, $V_{C,D}$?



Figure 26 Current Waveforms for 4:1 Ruthroff

 $-2V+V+V_{AB}+V_{L}=0$ VL was already determined above to be (1/2) V. $V=V_{AB}+\frac{1}{2}V$ $V_{AB}=\frac{1}{2}V$

9:1 Unbalanced to Unbalanced from ARRL Handbook

Figure 14.57 2007 ARRL Handbook



Figure 27 9:1 Balun

This next TLT posed multiple simulation problems in that fundamentally the TLTs, as mentioned multiple times before, is based on transmission lines not flux linkages in normal transformers. When setting the coupling coefficients between all inductors equal, i.e. L1-L2, L1-L3, L1-L4, L2-L3, L3-L4, etc., the Simetrix simulator "choked" due to numerical instabilities. For this 9:1 TLT to function as expected, we need to see 3X as much current at the input as at the output in R₂.

As before, a simplistic approach to analyzing the currents is done and shown below. L_1 and L_2 are considered "coupled" as are L3 and L4.



Figure 4 Current Analysis of 9:1 Balun

In the final analysis to the left we see that indeed there is 3X the current in the lower impedance input compared to the higher impedance output.

Transformer Background



When R_2 is small the ratio is essentially M/L_2 and when R_2 is very large the current i_2 becomes quite small (no surprise).

$$i_{1} = \frac{\begin{vmatrix} V & -j\omega M \\ 0 & R_{2} + j\omega L_{2} \end{vmatrix}}{\begin{vmatrix} (R_{1} + j\omega L_{1}) & -j\omega M \\ -j\omega M & (R_{2} + j\omega L_{2}) \end{vmatrix}} = \frac{R_{2} + j\omega L_{2}}{\Delta} \qquad i_{2} = \frac{\begin{vmatrix} R_{1} + j\omega L_{1} & V \\ -j\omega M & 0 \end{vmatrix}}{\begin{vmatrix} (R_{1} + j\omega L_{1}) & -j\omega M \\ -j\omega M & (R_{2} + j\omega L_{2}) \end{vmatrix}} = \frac{j\omega M}{\Delta}$$

¹ Transmission Line Transformers, American Radio Relay League, Jerry Sevick, 1987

² Modern Communication Circuits, Chapter 6, McGraw Hill, Jack Smith, 1986

³ Transmission Line Transformers by K5TRA, <u>http://k5tra.net/tech%20library/RF%20transformers/Transmission-Line%20Transformers.pdf</u>