

Handling Dependent Current Sources

A Prelude to Transistor Modeling

Jeffrey E. Crawford

1 Why Dependent Sources?

Early in the first class of an electrical engineering program, one is exposed to voltage and current sources. A battery is considered an *independent source* because its voltage is defined as V and nothing can influence that other than perhaps shorting its terminals. In a similar manner, one can have independent current sources which, as one would guess, deliver the same current no matter what the load (the components receiving the current) is except, in this case, an open circuit. A *dependent* source, whether voltage or current, has its respective voltage or current controlled by some other voltage or current in a circuit. Shortly, an example of this will be provided.

Dependent sources can be found in multiple places in circuits, but the first one a student is likely introduced to is in an *equivalent circuit* of a transistor. Depending on the type of model used for a BJT (bipolar junction transistor), the dependent source could be a voltage or current source. Diving into this additional detail is unnecessary for this discussion. Dependent voltage and current sources have their place in circuit theory and the analyst must understand how to deal with them.

2 Dependent Current Source Example

The remaining discussion centers around the simple circuit illustrated in Figure 1. This circuit has two resistors, one dependent current source of 24 ma, and a dependent current source whose current output is dependent, i.e, tied to the current labeled i_x . In this problem we wish to determine the values of i_6 and i_x . Another variable, v , is shown in the figure, but it is used only in an auxiliary sense as will soon be seen.

It is known from Kirchoff's current law that the net current at any node is zero, in other words there is a balance between currents entering a node and the currents leaving a node. While either convention is equally valid, here the currents entering the top node of the circuit are considered negative (-) currents and the currents leaving the top node are considered positive (+). With this convention the following equation may be written.

$$i_6 - 2i_x - 0.024 - i_x = 0 \quad (1)$$

The auxiliary variable v is now used to write two expressions for the currents a) i_6 and current b) i_x that passes through the 2 k Ω resistor.

$$i_6 = \frac{v}{6 \text{ k}\Omega} \quad i_x = \frac{-v}{2 \text{ k}\Omega} \quad (2)$$

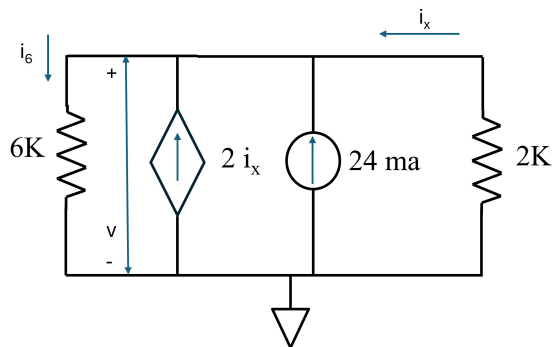


Figure 1: Circuit With Dependent and Independent Sources

These two equations are substituted into equation (1) giving the following intermediate result.

$$\frac{v}{6 \text{ k}\Omega} - 2 \left(\frac{-v}{2 \text{ k}\Omega} \right) - 0.024 + \frac{v}{2 \text{ k}\Omega} = 0 \quad (3)$$

Combining like terms gives

$$v \left(\frac{1}{6 \text{ k}\Omega} + \frac{1}{1 \text{ k}\Omega} + \frac{1}{2 \text{ k}\Omega} \right) = 0.024 \quad (4)$$

Performing the arithmetic determines that the auxiliary variable v is 14.4 volts. This voltage can then be used to calculate i_6 and i_x .

$$i_6 = \frac{v}{6 \text{ k}\Omega} = \frac{14.4}{6000} = 2.4 \text{ ma} \quad i_x = \frac{14.4}{2 \text{ k}\Omega} = \frac{14.4}{2000} = 7.2 \text{ ma} \quad (5)$$

It is good practice to see if these answers comply with Kirchoff's current law at the top node. Therefore we can write, using the same sign convention for the entering/exiting currents as before

$$2.4 \text{ ma} - 2 \times (-7.2 \text{ ma}) - 24 \text{ ma} - (-7.2 \text{ ma}) = 0 \quad (6)$$

Let's dissect this last equation (6) to alleviate any remaining concerns about signs for the individual currents.

- Current i_6 : $i_6 = + 2 \text{ ma}$ because it is leaving the top node
- dependent current: $2i_x$ is negative because it is entering. **However**, current i_x is itself negative, therefore the net current is $-2(-7.2 \text{ ma}) = 14.4 \text{ ma}$
- independent source: fixed at -24 ma because it is entering the node
- Current i_x : i_x is considered negative because it is entering the node and the voltage across its resistor from top to bottom is $-v$ or -14.4 V , therefore giving $-(-14.4/2 \text{ k}\Omega) = 7.2 \text{ ma}$.

It is very easy to become confused, mostly because the dependent variable i_x is defined in 'the wrong direction' of the resulting current flow, and it is used in the dependent current source as well.

Alternatively, a different look at the circuit may help your understanding. Looking at the right half of the circuit in Figure 1, it is possible to write the following simple equation (7). It can be said, therefore, that the left half of Figure 1 has a total current of 16.8 ma just like the right half, but flowing in the opposite direction.

$$i = 0.024 - \frac{14.4}{2 \text{ k}\Omega} = 16.8 \text{ ma} \quad (7)$$

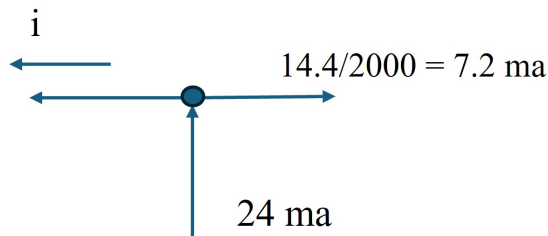


Figure 2: Circuit With Dependent and Independent Sources

Looking at both Figures 1 and 2, the current passing through the $6\text{ k}\Omega$ resistor is $14.4\text{V}/6\text{ k}\Omega = 2.4\text{ ma}$, therefore the current passing through the dependent current source is $16.8\text{ ma} - 2.4\text{ ma} = 14.4\text{ ma}$ flowing down out of the dependent source. Dividing by 2 corroborates the fact that $i_x = -7.1\text{ ma}$.